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Theory of Hard Diffraction

- I. Introduction
- II. Building block: BFKL
- III. $\gamma\gamma$ -scattering in e^+e^-
- IV. γp -scattering in DIS
- V. Theory: AGK, QCD-Reggeon Field Theory
- VI. pp-scattering

Some overlap with other lectures

I. Introduction

Why diffraction:

- always present: total cross section $\sigma_{tot} \sim \frac{1}{s} \text{Im} T(s, 0)$
- rapidly gaps low, high mass diffraction

- not small fraction



- understanding underlying event

Working definition:

- rapidity gap in final state
- color singlet exchange

Nonperturbative vs. pQCD:

- elastic scattering: probability that projectiles stay intact
 \rightarrow high energy manifestation of binding forces

- Regge pole model: 

$$T_{AB} \sim i s \beta_A s^{\alpha(t)} \beta_B, \quad \alpha(t) = 1 + \alpha' t, \quad \beta \sim e^{r^2 t}$$

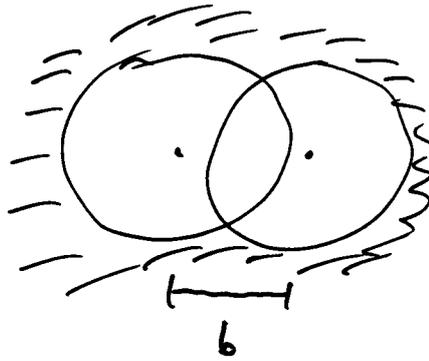
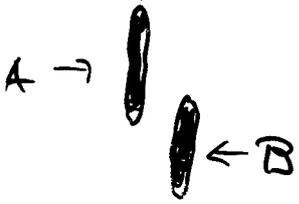
$$= i s \int d^2 \vec{b} e^{i \vec{q} \cdot \vec{b}} f_{AB}(s, \vec{b}) \quad t = -q^2$$

$$f_{AB} \sim e^{-b^2/R^2}, \quad R^2 = r_A^2 + r_B^2 \sim \alpha' \ln s$$

\rightarrow exponential decrease at large b^2

(effective Pomeron² grows as $\ln s$ ($\ln^2 s$))

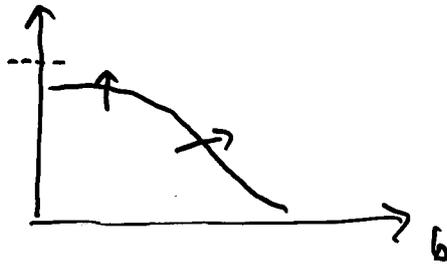
Transverse plane:



$$R^2 = r_A^2 + r_B^2 + \alpha' l_{us}$$

Diffusion
wee particles

$f(s, \vec{b})$ in reality: $|f(s, b)| \leq 1$ unitarity bound



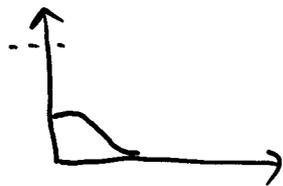
growth and expansion

In perturbation theory of massless vector particles:

- leading-log does not respect unitarity bound

$$T_{BKEL}(s, t) \sim s^{1 + \omega_{BKEL}} \quad (\text{weak } t\text{-dependence})$$

- decrease at large b weak, radius grows too fast



Conclusion: with present technology (pQCD + corrections)

- start from small-transverse size regions
- at larger distances: need to model, effective theories, ...

Attitude taken in this talk:

- start (over $p(Q)$)
- analyse region of validity
- emphasis on transition to $up(Q)$

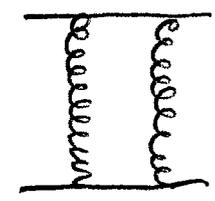
Intention:

- elementary, slow
- few new results

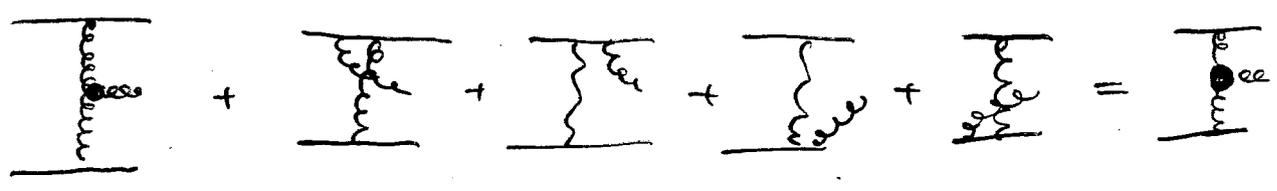
II. Building Block: BFKL

Color singlet in t -channel:

2 gluon exchange

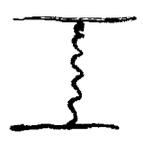


Particle production: multiperipheral kinematics



effective production vertex
violate energy momentum conservation

Gluon reggeization:

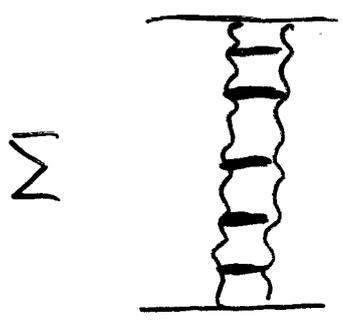


$$T \sim g^2 s^{\alpha(t)}$$

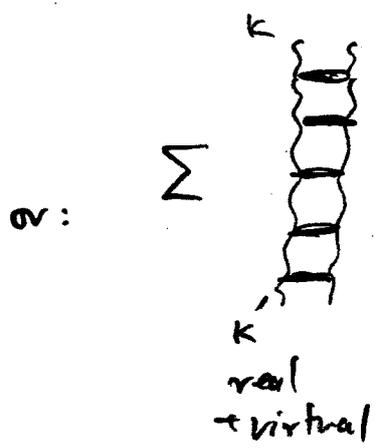
$$\alpha_g(t) = 1 + t \cdot \frac{N_c g^2}{2} \int \frac{d^2k}{(2\pi)^3} \frac{1}{k^2} \frac{1}{(q-k)^2}$$

$$-q^2 = t$$

BFKL ladders:



real + reggeized gluons



$$\Sigma = G_\omega(k, q-k; k', q-k')$$

$$= \text{diagram with a circle vertex}$$

$$G_\omega(k, q-k; k', q-k') = (2\pi)^3 \delta(k-k') \frac{1}{k^2 (q-k)^2}$$

$$+ \frac{1}{k^2 (q-k)^2} \int \frac{d^2k''}{(2\pi)^3} K_{BFKL}(k, k'', q) G_\omega(k'', q-k''; k', q-k')$$

Most important results:

- growth with energy $T \sim \omega^{\omega_{BFKL}}$

$$\omega_{BFKL} = \frac{4 \ln 2 N_c \alpha_s}{\pi}$$

- diffusion (see below)

- characteristic function $\chi(\nu, u) \leftrightarrow$ anomalous dimensions

Bootstrap in color octet exchange:

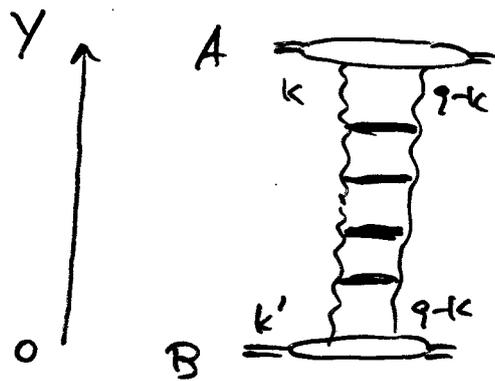
$$\sum_{\mathcal{P}_A} \text{[Diagram of a vertical chain of four vertices with wavy lines and a label } \mathcal{P}_A \text{ below]} = \text{[Diagram of a vertical wavy line with a label } [\alpha_g - 1] \text{ to its right]}$$

Unitarity: $\frac{1}{2} T^\dagger T = T - T^\dagger$

$$\frac{1}{2} \sum_n |T_{2+n}|^2 d\mathcal{R}_n = \mathcal{R}_n T_{2+2}$$

$$\text{[Diagram of a vertical chain of four vertices with wavy lines]} \quad \mathcal{R}_n \text{ [Diagram of a vertical wavy line]}$$

Realistic scattering amplitude: needs impact factors
heavy photon, quark-quark



$$T = \int \frac{d\omega}{2\pi i} \int \frac{d^2k d^2k'}{(2\pi)^{3-2}} \left(\frac{s}{kk'}\right)^{i\omega} \xi_\omega$$

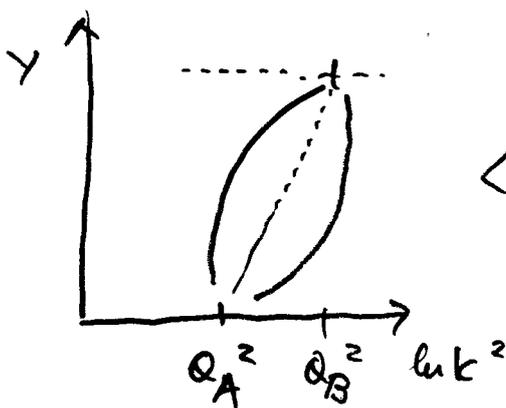
$$\phi_A(k, q-k) G_\omega(k, q-k; k', q-k') \phi_B(k', q-k')$$

$$\phi(k, q-k) \rightarrow 0$$

$k \rightarrow 0$
or $q-k \rightarrow 0$

"good properties"
(gauge invariance, color transparency)
Regge factorization

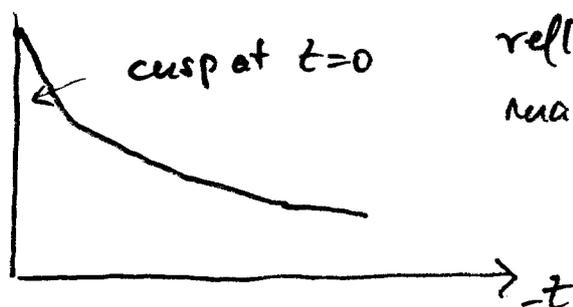
Diffusion:
($t=0$)



$$\langle (\Delta \ln k^2)^2 \rangle = \text{const. } Y$$

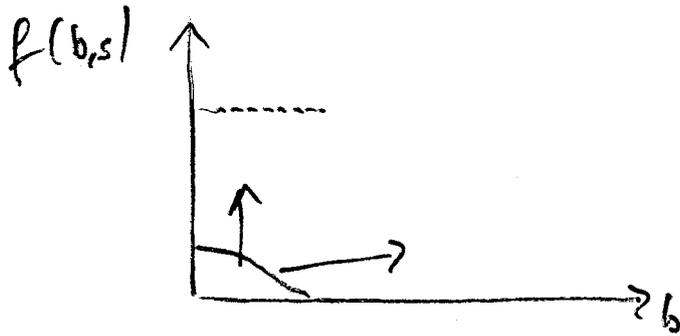
transverse radius grows faster than $\ln s$:
massless gluons

Elastic cross section $\frac{d\sigma}{dt}$:



reflects singularity of massless gluons

Summary: b -profile:



- fixed b : grows beyond unitarity limit
- at large b : radius grows too fast

Unitarization: corrections to $\overline{\text{BFCL}}$

- stop the growth with energy at fixed b :
 \rightarrow nonlinear evolution
- modify large- b behavior:
 beyond perturbation theory

NLO:

- running α_s
- energy momentum conservation
 (phenomenological treatment of NLO effects)
- resummation